

Полярные координаты

Задан закон движения точки в полярных координатах: $\rho = \rho(t)$ (в метрах), $\varphi = \varphi(t)$. В указанный момент времени найти скорость и ускорение точки в полярных, декартовых и естественных координатах.

Кирсанов М.Н. Решебник. Теоретическая механика/Под ред. А. И. Кириллова.– М.:ФИЗМАТЛИТ, 2002.– 384 с. (с.144.)

Задача 4.1

3

$$r = \frac{16 \sin^2(t/10)}{\cos(t/10)},$$

$$\varphi = \frac{t}{10}, \quad t = 8 \text{ c.}$$

Задача 4.3

3

$$r = \frac{9}{5}t - \frac{90}{t},$$

$$\varphi = \arccos(t/10), \quad t = 7 \text{ c.}$$

Задача 4.5

3

$$r = 10(t/5)^5,$$

$$\varphi = (t/5)^5, \quad t = 4 \text{ c.}$$

Задача 4.7

3

$$r = 24/(1 + t/20),$$

$$\varphi = \arccos(t/12), \quad t = 9 \text{ c.}$$

Задача 4.9

3

$$r = 10 \cos(t/6) + 11,$$

$$\varphi = t/6, \quad t = 4 \text{ c.}$$

Задача 4.11

3

$$r = -\frac{19 \cos(2t/13)}{\cos(t/13)},$$

$$\varphi = \frac{t}{13}, \quad t = 10 \text{ c.}$$

Задача 4.13

3

$$r = 27/(1 + t/8),$$

$$\varphi = \arccos(t/8), \quad t = 5 \text{ c.}$$

Задача 4.15

3

$$r = \frac{14 \sin^2(t/11)}{\cos(t/11)},$$

$$\varphi = \frac{t}{11}, \quad t = 8 \text{ c.}$$

Задача 4.17

3

$$r = \frac{12 \sin^2(t/8)}{\cos(t/8)},$$

$$\varphi = \frac{t}{8}, \quad t = 6 \text{ c.}$$

Задача 4.2

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$$r = 28/(1 + 3t/25),$$

$$\varphi = \arccos(t/5), \quad t = 2 \text{ c.}$$

Задача 4.4

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$$r = 26/(1 + \frac{4}{5}t),$$

$$\varphi = \arccos(t/5), \quad t = 4 \text{ c.}$$

Задача 4.6

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$$r = 2t \sin(t/11),$$

$$\varphi = t, \quad t = 1 \text{ c.}$$

Задача 4.8

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$$r = \frac{2}{3}t - \frac{48}{t},$$

$$\varphi = \arccos(t/12), \quad t = 10 \text{ c.}$$

Задача 4.10

3

$$r = \frac{24}{1+\cos(t/6)},$$

$$\varphi = \frac{t}{6}, \quad t = 2 \text{ c.}$$

Задача 4.12

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$$r = 19 \cos^2(\pi t/16),$$

$$\varphi = \cos^2(\pi t/16), \quad t = 10 \text{ c.}$$

Задача 4.14

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$$r = 15e^{-t/8},$$

$$\varphi = e^{t/8}, \quad t = 5 \text{ c.}$$

Задача 4.16

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$$r = 14e^{-t/9},$$

$$\varphi = e^{t/9}, \quad t = 5 \text{ c.}$$

Задача 4.18

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$$r = 7t \sin(t/11),$$

$$\varphi = t, \quad t = 6 \text{ c.}$$

Задача 4.19

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$$r = 80(1 - (t/10)^2)/t,$$
$$\varphi = \arccos(t/10), \quad t = 8 \text{ c.}$$

Задача 4.21

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$$r = 108(1 - (t/12)^2)/t,$$
$$\varphi = \arccos(t/12), \quad t = 10 \text{ c.}$$

Задача 4.23

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$$r = 10 \cos(t/7) + 12,$$
$$\varphi = t/7, \quad t = 4 \text{ c.}$$

Задача 4.25

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$$r = -\frac{17 \cos(2t/5)}{\cos(t/5)},$$
$$\varphi = \frac{t}{5}, \quad t = 1 \text{ c.}$$

Задача 4.27

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$$r = 42(1 - (t/7)^2)/t,$$
$$\varphi = \arccos(t/7), \quad t = 5 \text{ c.}$$

Задача 4.29

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$$r = 25/(1 + t/8),$$
$$\varphi = \arccos(t/8), \quad t = 6 \text{ c.}$$

Задача 4.20

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$$r = \frac{28}{1+0.4 \cos(t/3)},$$
$$\varphi = \frac{t}{3}, \quad t = 1 \text{ c.}$$

Задача 4.22

3

$$r = -\frac{15 \cos(2t/5)}{\cos(t/5)},$$
$$\varphi = \frac{t}{5}, \quad t = 3 \text{ c.}$$

Задача 4.24

3

$$r = 10t/9 + 9,$$
$$\varphi = \arccos(t/9), \quad t = 5 \text{ c.}$$

Задача 4.26

3

$$r = \frac{24}{1+\cos(t/5)},$$
$$\varphi = \frac{t}{5}, \quad t = 2 \text{ c.}$$

Задача 4.28

3

$$r = 9e^{t/5},$$
$$\varphi = t, \quad t = 4 \text{ c.}$$

Задача 4.30

3

$$r = 23/(1 + t/8),$$
$$\varphi = \arccos(t/8), \quad t = 7 \text{ c.}$$

Полярные координаты

№	ρ	$\dot{\rho}$	φ	$\dot{\varphi}$	v_ρ	v_φ	v	v_x	v_y	Кривая
1	11.818	3.512	0.800	0.100	3.512	1.182	3.706	1.599	3.343	Циссоида
2	22.581	-2.185	1.159	-0.218	-2.185	-4.928	5.390	3.642	-3.974	Эллипс
3	-0.257	3.637	0.795	-0.140	3.637	0.036	3.637	2.520	2.622	Строфоида
4	6.190	-1.179	0.644	-0.333	-1.179	-2.063	2.377	0.295	-2.358	Гипербола
5	3.277	4.096	0.328	0.410	4.096	1.342	4.310	3.446	2.589	Архимедова спираль
6	0.182	0.363	1.000	1.000	0.363	0.182	0.406	0.043	0.403	
7	16.552	-0.571	0.723	-0.126	-0.571	-2.085	2.162	0.951	-1.942	Эллипс
8	1.867	1.147	0.586	-0.151	1.147	-0.281	1.181	1.111	0.399	Строфоида
9	18.859	-1.031	0.667	0.167	-1.031	3.143	3.308	-2.754	1.833	Улитка Паскаля
10	12.340	0.346	0.333	0.167	0.346	2.057	2.085	-0.346	2.057	Парабола
11	-0.855	4.003	0.769	0.077	4.003	-0.066	4.003	2.922	2.737	Строфоида
12	2.782	2.638	0.146	0.139	2.638	0.386	2.666	2.553	0.767	Архимедова спираль
13	16.615	-1.278	0.896	-0.160	-1.278	-2.661	2.952	1.278	-2.661	Парабола
14	8.029	-1.004	1.868	0.234	-1.004	1.875	2.127	-1.499	-1.509	Гиперболическая спираль
15	8.284	2.363	0.727	0.091	2.363	0.753	2.480	1.264	2.133	Циссоида
16	8.033	-0.893	1.743	0.194	-0.893	1.556	1.793	-1.380	-1.146	Гиперболическая спираль
17	7.620	2.932	0.750	0.125	2.932	0.953	3.083	1.496	2.696	Циссоида
18	21.790	6.896	6.000	1.000	6.896	21.790	22.855	12.710	18.995	
19	3.600	-2.050	0.644	-0.167	-2.050	-0.600	2.136	-1.280	-1.710	Циссоида
20	20.320	0.643	0.333	0.333	0.643	6.773	6.804	-1.608	6.611	Эллипс
21	3.300	-1.830	0.586	-0.151	-1.830	-0.497	1.896	-1.250	-1.426	Циссоида
22	-6.586	5.875	0.600	0.200	5.875	-1.317	6.020	5.592	2.230	Строфоида
23	20.411	-0.773	0.571	0.143	-0.773	2.916	3.017	-2.227	2.035	Улитка Паскаля
24	14.556	1.111	0.982	-0.134	1.111	-1.945	2.240	2.235	-0.157	Улитка Паскаля
25	-15.977	2.054	0.200	0.200	2.054	-3.195	3.799	2.648	-2.724	Строфоида
26	12.493	0.506	0.400	0.200	0.506	2.499	2.549	-0.506	2.499	Парабола
27	4.114	-2.537	0.775	-0.204	-2.537	-0.840	2.673	-1.224	-2.376	Циссоида
28	20.030	4.006	4.000	1.000	4.006	20.030	20.427	12.540	-16.124	Логарифмическая спираль
29	14.286	-1.020	0.723	-0.189	-1.020	-2.700	2.886	1.020	-2.700	Парабола
30	12.267	-0.818	0.505	-0.258	-0.818	-3.167	3.271	0.818	-3.167	Парабола

Nº	$\ddot{\rho}$	$\ddot{\varphi}$	W_ρ	W_φ	a	W_x	W_y	$ W_\tau $	W_n
1	0.828	0.000	0.710	0.702	0.999	-0.009	0.999	0.897	0.439
2	0.423	-0.021	-0.652	0.484	0.813	-0.705	-0.404	-0.178	0.793
3	-0.525	-0.019	-0.520	-1.014	1.139	0.360	-1.081	-0.530	1.008
4	0.449	-0.148	-0.239	-0.131	0.272	-0.112	-0.248	0.232	0.142
5	4.096	0.410	3.546	4.698	5.886	1.846	5.589	4.833	3.360
6	0.361	0.000	0.179	0.725	0.747	-0.514	0.543	0.485	0.568
7	0.039	-0.018	-0.223	-0.154	0.271	-0.066	-0.263	0.208	0.175
8	-0.096	-0.034	-0.138	-0.410	0.432	0.111	-0.418	-0.037	0.431
9	-0.218	0.000	-0.742	-0.344	0.818	-0.371	-0.729	-0.095	0.812
10	0.186	0.000	-0.157	0.115	0.195	-0.186	0.058	0.088	0.174
11	0.611	0.000	0.616	0.616	0.871	0.015	0.871	0.606	0.626
12	1.036	0.055	0.982	0.884	1.322	0.843	1.018	1.100	0.733
13	0.197	-0.021	-0.229	0.068	0.239	-0.197	-0.136	0.038	0.236
14	0.125	0.029	-0.312	-0.234	0.391	0.316	-0.230	-0.059	0.386
15	0.487	0.000	0.418	0.430	0.600	0.027	0.599	0.529	0.282
16	0.099	0.022	-0.202	-0.173	0.266	0.205	-0.169	-0.049	0.261
17	0.838	0.000	0.719	0.733	1.027	0.027	1.027	0.910	0.475
18	0.908	0.000	-20.882	13.792	25.025	-16.197	19.077	6.848	24.070
19	0.313	-0.037	0.213	0.550	0.590	-0.160	0.568	-0.358	0.468
20	0.660	0.000	-1.598	0.429	1.654	-1.650	-0.117	0.276	1.631
21	0.216	-0.034	0.141	0.439	0.461	-0.125	0.444	-0.251	0.386
22	2.398	0.000	2.661	2.350	3.550	0.870	3.442	2.083	2.875
23	-0.172	0.000	-0.588	-0.221	0.628	-0.375	-0.504	-0.063	0.625
24	0.000	-0.012	-0.260	-0.471	0.538	0.247	-0.478	0.280	0.459
25	2.084	0.000	2.723	0.822	2.844	2.505	1.346	0.781	2.735
26	0.281	0.000	-0.219	0.203	0.298	-0.281	0.101	0.155	0.255
27	0.672	-0.043	0.501	0.861	0.996	-0.245	0.965	-0.746	0.660
28	0.801	0.000	-19.229	8.012	20.831	18.632	9.315	4.085	20.427
29	0.146	-0.040	-0.364	-0.193	0.412	-0.146	-0.386	0.309	0.273
30	0.109	-0.120	-0.709	-1.056	1.272	-0.109	-1.267	1.199	0.422